Name:

Math 10a
October 16, 2014
Quiz #5

1. Do the following series converge or diverge? Justify your answer.

   (a) (2 points)

   $$1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \cdots + \frac{1}{n!} + \cdots$$

   Ratio test: $\left|\frac{1/(n+1)!}{1/n!}\right| = \left|\frac{1}{n+1}\right| \to 0$ so the series converges.

   (b) (2 points)

   $$\sum_{k=0}^{\infty} \frac{(-1)^k k}{(2k)!}.$$

   Ratio test:

   $$\left|\frac{\frac{(-1)^{k+1}(k+1)}{(2(k+1))!}}{\frac{(-1)^k k}{(2k)!}}\right| = \left|\frac{(k+1)(2k)!}{k(2k+2)!}\right| = \left|\frac{k+1}{(2k+2)(2k+1)k}\right| \to 0$$

   so the series converges.

2. (1 point) Write down a series of rational numbers converging to e.

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

so

$$e = e^1 = \sum_{k=0}^{\infty} \frac{1}{k!}.$$

3. (3 points) What is the radius of convergence of the Taylor series of $\frac{1}{1-x}$ centered at $x = 0$?

   Ratio test:

$$\left|\frac{x^{k+1}}{x^k}\right| = |x|$$

   so the series converges for $|x| < 1$ and for diverges for $|x| > 1$. Hence the radius of convergence is 1.

4. (2 points) What is the area of the region bounded by the curve $y = x^2$ and the line $y = 1$? You are welcome to use the fact that $\int_0^1 x^2 dx = \frac{1}{3}$.

$$2 - \frac{1}{3} - \frac{1}{3} = \frac{4}{3}.$$